Application of NSGA in the Batching of Regenerated Polyester Staple Fiber Production

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Abstract

As the first step of regeneration textile production, the influence of blending quality on the quality and product cost of hollow polyester staple fiber is very important. This model is based on the theories of material balance, energy conservation, the mathematical programming theory as well as physical and chemical reaction in the process of melt spinning. On the premise of considering the general constraint, the inherent relationship between physical changes in the production of polyester staple fibers and the constraints of technological requirements in the production of regenerated polyester staple fibers are included. Meanwhile, using the lowest cost of ingredients as the objective mathematical programming function, and the model is solved by NSGA. The ingredients table was output through the interface of logistics management information system to simulate the ingredients, which provided ideas for the research on the ingredients of waste plastics regeneration spinning.

Keywords: Waste Plastics, Regeneration Textile, Mathematical Model, NSGA.

1. Introduction

Ingredients are the first important step in the production of regenerated polyester staple fibers. Various kinds of PET bottles from different sources are used as basic materials, and the reasonable proportion of bottles is used to achieve the purpose of producing polyester staple fibers. In the past production process, batching often relies on the experience of batching personnel, even using manual calculation is very cumbersome, it is difficult to find the proportion that not only meets the process conditions, but also makes the unit raw material consumption and cost of the product the lowest. The objective of this paper is to achieve the rational utilization of resources by reforming the way of blending and automatically calculating the results of blending by computer, so as to reduce the production cost of regenerated polyester staple fibers and obtain the greatest economic benefits by optimizing the blending ratio.

Literature [1] through the establishment of a clear linear programming model based on the basic technological constraints and the use of simplicity algorithm to solve the mathematical model, the batching optimization is basically achieved. However, this batching method does not consider the actual operation, because of weather and burning reasons, it is often hoped that the content of some indicators in the batching results will deviate from the specification limit as far as possible, resulting in the content of each component of waste PET in the batching results at the end of the allowable range, which is not expected to be seen in the production process. Literature [2,4] proposed a linear programming model based on fuzzy constraints. Literature [5] based on linear regression theory, analyzed the influence of mixing ratio on energy consumption to optimize mixing ratio. However, these batching models only take the lowest batching cost as the objective function, and do not fully consider the constraints of technological conditions, which obviously has a gap with the actual production of enterprises.

In this paper, by establishing an objective function based on the lowest cost of PET bottle filling, and combining with the classical multi-objective optimization algorithm NSGA, the constrained multi-objective mathematical model is solved and calculated, and a set of Pareto optimal solution sets is generated to provide decision support for actual batching production, which improves the flexibility and practicability of the batching model. The batching personnel can according to themselves. Subjective judgment and objective situation make different estimates of the level of constraint satisfaction, so as to obtain the satisfactory solution under the corresponding circumstances, which makes the batching process more flexible.

2 PET Bottle Batching Model

2.1 Objective function of pet bottle batching model

With m kinds of raw materials for production of certain polyester staple fiber used in control of the polyester staple fiber of n items of physical indicators. The amount of raw materials, with each person respectively, \( x_1, x_2, \ldots, x_m \) is the decision variable, it is clear that all decision variables satisfy the non-negative condition \( x_i \geq 0, i = 1, 2, \ldots, m \).

The objective function of this model is to minimize the cost of batching.
\[ \min W = \sum_{i=1}^{m} z_i x_i \]  

(1)

Where: \( z_i \) - \( i \)-kinds of raw material prices(yuan/kg); \( W \) - Total cost of ingredients/yuan; \( x_i \) - \( i \)-kinds of PET bottle.

2.2 Constraint condition

The constraints of this model mainly include two aspects: basic constraints and process constraints.

The basic constraints are mainly considered from three aspects: component constraints, permissible dosage constraints and total volume constraints of PET bottles.

(1) Component constraint: the amount of raw materials will change the physical state of waste PET, so we can adjust the melting state of PET by adjusting the amount of raw materials to achieve the production conditions of polyester staple fiber. According to the principle of material balance, the following conclusions are obtained:

\[ \sum_{j=1}^{q} (x_i \times b_{ij} \times f_j) \geq QE_{\min j} \]

(\( j = 1, 2, ..., q \))

\[ \sum_{j=1}^{q} (x_i \times b_{ij} \times f_j) \leq QE_{\max j} \]

(2) Maximum stock of each PET bottle is used as the maximum allowable quantity of each PET bottle. The constraints are as follows.

\[ 0 \leq x_i \leq K_i, i = 1, 2, ..., v \]

(3) Total constraint. The proportioning of raw materials according to the production of polyester staple fibers. \( v \)-kinds of PET bottle determines the production of polyester staple fiber’s index of \( q \) component, therefore, it is necessary to restrict the total amount according to the weight of material after mixing.

\[ M = \sum_{j=1}^{q} \sum_{i=1}^{m} (b_{ij} x_i) \leq Q \]

(4)

Where: \( Q \) is production line capacity. Index of \( q \) component: PVC, Si, S, P, Mn, Fe.

The main purpose of melt spinning is to remove PVC, phosphorus, sulfur, carbon, gas and impurities. The main influencing factors are temperature, alkalinity and oxygen content. In this paper, alkalinity constraints are mainly considered based on the constraints of process conditions. The most commonly used alkalinity expression method [6] is as follows:

\[ R = \frac{\% Ca + \% MgO}{\% SiO_2} \]

(5)

The alkalinity constraints of melted PET are as follows:

\[ R_{\min} \leq R = \frac{\% CaO + \% MgO}{\% SiO_2} \leq R_{\max} \]

(6)

Where: \((\% SiO_2)\) is Content of \( SiO_2 \) in PET(From the Chemical Equilibrium Principle in Reaction); \( R_{\min} \) - limit of alkalinity.

3. NSGA Algorithms and Model Solution

In this paper, NSGA algorithm is applied to solve the optimization model of PET bottle structure. The steps are as follows[8]: (1) initial population with random population size of 200- \( P_0 \); (2) Computational population- \( P \); object function values and non-inferior ranking; (3) Generation \( Q_i \) of offspring by genetic manipulation of population \( P \); (4) Population merging \( R_i = P_i \cup Q_i \); Non-inferior ranking and congestion distance calculation for individuals in \( R_i \); Computation of Crowding Degree Distance; (5) Select and retain elite...
individuals by using Championship rules, Selecting the first \( N \) individuals as the paternal population- \( P_{t+1} \);

(6) Circulation of execution procedures and negotiation of termination conditions, judgment according to the maximum genetic algebra set in advance, if \( t > 2000 \), algorithm termination.

According to the constraint processing technology based on evolutionary algorithm proposed in reference [8], equality constraints are transformed into inequality constraints.

\[
|h(x)| - \delta \leq 0
\]  

(7)

Where: \( \delta \) is tolerance Value of Equality Constraints. Generally, the smallest positive number is chosen. In addition, the extent to which individuals \( x \) in a group violate \( j \) constraint can be expressed by the following formula:

\[
G_j(x) = \begin{cases} 
\max(x, g_j(x)) \\
\max(0, h_j(x) - \delta)
\end{cases}
\]  

(8)

Where: \( i = 1, 2, \ldots, q \) : \( j = 1, 2, \ldots, p \).

\[
G(x) = \sum_{j=1}^{p} G_j(x)
\]  

(9)

Where: \( G(x) \) is the extent to which all constraints in the problem are violated for individual \( x \).

Based on the above analysis, the following penalty functions are constructed:

\[
\min F(x) = f(x) + u \sum_{j=1}^{p} G_j(x)^2
\]  

(10)

\( u \) is the penalty coefficient in the formula. By penalty function, processing equality constraints and inequality constraints, Converting the problem into an unconstrained objective problem and solving it with NSGA II algorithm.

Firstly, the formulas (1) and (2) are transformed into linear programming problems under penalty functions. For general mathematical programming, we know that all constraints can be transformed into inequality constraints only \((\leq)\). For the inequality constraints of \((\geq)\), the inequality constraints with \((\leq)\) are simply transformed into inequality constraints with \((\leq)\) by multiplying (-1) on both sides of the inequality.

\[
\max W = - \sum_{i=1}^{n} c_i x_i
\]  

(11)

\[
\sum_{j=1}^{2m} (x_i \times b_j) \leq b_i (i = 1, 2, \ldots, 2m)
\]  

(12)

The constraints of \( 2m \) in formula (12) expressed by \( \leq \) are rewritten to the following formula:

\[
\mu_D \left( \sum_{j=1}^{m} (x_j \times b_j) \right) = \begin{cases} 
1, & \sum_{j=1}^{m} (x_j \times b_j) \leq b_i \\
1 - \lambda_i \left( \sum_{j=1}^{m} (x_j \times b_j) - b_i \right), & b_i < \sum_{j=1}^{m} (x_j \times b_j) \leq b_i + \lambda_i \\
0, & \sum_{j=1}^{m} (x_j \times b_j) \geq b_i + \lambda_i
\end{cases}
\]  

(13)

Where \( \lambda_i (i = 1, 2, \ldots, m) \) is a given non-negative number and is called a scaling index. In order to solve the problem, the target value needs to be Fuzzy. Suppose \( Z_a \leq Z \leq + \lambda_a \), where \( \lambda_a > 0 \),

\[
\mu_M (x) = \begin{cases} 
0, & \sum_{j=1}^{n} (C_j x_j) < Z_0 \\
\frac{1}{\lambda_0} \left( \sum_{j=1}^{n} (C_j x_j - Z_0) \right), & Z_0 \leq \sum_{j=1}^{n} (C_j x_j) < Z_0 + \lambda_i \\
1, & Z_0 + \lambda_i \leq \sum_{j=1}^{n} C_j x_j
\end{cases}
\]  

(14)
Among them, $Z_0$ and $\lambda_0$ values are determined according to practical problems. Formulas (1) and (2) are transformed into the following models:

$$\max \mu_M (x) \lambda_\mu (x) \quad (15)$$

According to the relevant theorems in mathematics, the above formula can be converted into the following linear programming problems:

$$\begin{align*}
\text{max } g &= \omega \\
1 - \frac{1}{\lambda_i} (\sum_{j=1}^{n} a_j x_j - b_i) &\geq \omega, \quad i = 1, 2, ..., m \\
s.t. \frac{1}{\lambda_0} (\sum_{j=1}^{n} C_j x_j - Z_0) &\geq \omega \\
\omega &\leq 1 \\
\omega &\geq 0, x_1, x_2, ..., x_n \geq 0
\end{align*} \quad (16)$$

After finishing the top mathematical operations:

$$\begin{align*}
\sum_{j=1}^{n} a_j x_j + \lambda_i \omega &\leq b_i + \lambda_i, \quad i = 1, 2, ..., m \\
s.t. \sum_{j=1}^{n} C_j x_j - \lambda_0 \omega &\geq Z_0 \\
\omega &\leq 1 \\
\omega &\geq 0, x_1, x_2, ..., x_n \geq 0
\end{align*} \quad (17)$$

4. Calculation and Analysis of Batching Optimization Model

Take a chemical fibre factory as an example, 12 kinds of raw materials in stock (Purified material that has passed primary treatment of raw material), price is converted by deduction of loss from primary materials. The objective of batching is to satisfy the content of five substances. NSGA II algorithm was used to calculate the ingredients, and the results were as follows: Table 1 and Table 2.

<table>
<thead>
<tr>
<th>No.</th>
<th>Name</th>
<th>PET</th>
<th>PVC</th>
<th>Paper head and cap</th>
<th>Impurities</th>
<th>Moisture</th>
<th>Price (yuan/t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A-paper-covered white of the original Bottle</td>
<td>94.98</td>
<td>0.02</td>
<td>0.3</td>
<td>2.2</td>
<td>2.5</td>
<td>7500</td>
</tr>
<tr>
<td>2</td>
<td>B-paper-covered white of the original Bottle</td>
<td>94.90</td>
<td>0.1</td>
<td>0.2</td>
<td>2.4</td>
<td>2.4</td>
<td>7385</td>
</tr>
<tr>
<td>3</td>
<td>C-paper-covered white of the original Bottle</td>
<td>94.20</td>
<td>0.4</td>
<td>0.3</td>
<td>2.5</td>
<td>2.6</td>
<td>7524</td>
</tr>
<tr>
<td>4</td>
<td>A grade white block</td>
<td>96.80</td>
<td>0.1</td>
<td>0.2</td>
<td>2.1</td>
<td>2.5</td>
<td>8054</td>
</tr>
<tr>
<td>5</td>
<td>B grade white blocks</td>
<td>94.00</td>
<td>0.5</td>
<td>0.3</td>
<td>2.5</td>
<td>2.5</td>
<td>8000</td>
</tr>
<tr>
<td>6</td>
<td>C grade white blocks</td>
<td>98.26</td>
<td>0.04</td>
<td>0.1</td>
<td>1.6</td>
<td>2.5</td>
<td>8800</td>
</tr>
<tr>
<td>7</td>
<td>A grade white foam</td>
<td>95.00</td>
<td>0.1</td>
<td>0.2</td>
<td>0.9</td>
<td>2.8</td>
<td>8020</td>
</tr>
<tr>
<td>8</td>
<td>B grade white foam</td>
<td>93.80</td>
<td>0.5</td>
<td>0.3</td>
<td>2.9</td>
<td>2.9</td>
<td>7400</td>
</tr>
<tr>
<td>9</td>
<td>C grade white foam</td>
<td>97.70</td>
<td>0.1</td>
<td>0.1</td>
<td>1.1</td>
<td>2.1</td>
<td>7650</td>
</tr>
<tr>
<td>10</td>
<td>A grade white silk</td>
<td>96.30</td>
<td>0.5</td>
<td>0.1</td>
<td>1.1</td>
<td>2.2</td>
<td>7800</td>
</tr>
<tr>
<td>11</td>
<td>B grade white silk</td>
<td>94.91</td>
<td>0.09</td>
<td>0.2</td>
<td>2.4</td>
<td>2.4</td>
<td>7580</td>
</tr>
</tbody>
</table>

Table 2. Three-dimensional hollow with silicone (7D×51MM)

<table>
<thead>
<tr>
<th>Name</th>
<th>PET</th>
<th>PVC</th>
<th>Paper head and cap</th>
<th>Impurities</th>
<th>Moisture</th>
<th>Price (yuan/t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7D×51—YY—A</td>
<td>95.1-96.3</td>
<td>0.07-0.09</td>
<td>&lt;0.19</td>
<td>1.8-2.2</td>
<td>&lt;2.5</td>
<td></td>
</tr>
</tbody>
</table>

The distribution of Pareto optimal solution obtained by NSGA II algorithm is more uniform and reasonable than that of fuzzy programming algorithm. The two objective functions of each set of solutions calculated by the two algorithms correspond to one point on the Pareto front in Figure 1.
In Figure 1, the abscissa represents the cost per ton of waste plastics and the ordinate represents the value of energy consumption. As can be seen from the figure, when the iteration stops, the burden cost of NSGA II algorithm is lower. The Pareto optimum solutions of the batching optimization problem of regenerated polyester staple fibers obtained by two algorithms are listed in Table 3.

Table 3. Actual burden and optimized burden compare (The preparation of 100 tons of materials)

<table>
<thead>
<tr>
<th>Name</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Artificial ingredients</td>
<td>74.61</td>
<td>7.46</td>
<td>2.57</td>
<td>8.34</td>
<td>0</td>
<td>0.24</td>
<td>4.30</td>
<td>0</td>
<td>0.32</td>
<td>1.23</td>
<td>0.93</td>
<td>0</td>
</tr>
<tr>
<td>NSGAII</td>
<td>70.63</td>
<td>5.12</td>
<td>1.57</td>
<td>5.34</td>
<td>0.56</td>
<td>9.24</td>
<td>2.30</td>
<td>0.90</td>
<td>0.62</td>
<td>0.23</td>
<td>0.93</td>
<td>2.56</td>
</tr>
<tr>
<td>FP</td>
<td>72.60</td>
<td>2.06</td>
<td>4.57</td>
<td>6.34</td>
<td>0.98</td>
<td>6.24</td>
<td>4.39</td>
<td>0.67</td>
<td>0.82</td>
<td>1.03</td>
<td>0.23</td>
<td>1.67</td>
</tr>
</tbody>
</table>

In order to verify the diversity of the solution set distribution and convergence of the two algorithms, the convergence and diversity information of the two algorithms are evaluated by using the distance approach method and the population diversity evaluation method based on the aggregation distance D. The simulation results are shown in figs. 2 and 3.
The simulation results show that the convergence of the two algorithms is faster and the population diversity is better. But for this model, the convergence of NSGA II algorithm is stronger than that of FP algorithm, and the population diversity is better. In addition, the success rate of finding the optimal solution of the model through 50 independent experiments of NSGA II algorithm and FP algorithm is 91.2% and 87.6% respectively, and the solving time is 201 seconds and 264 seconds respectively, which shows that NSGA II algorithm is better than FP in the accuracy and efficiency of solving the model. Based on the calculation results of the 200th generation run by MATLAB and the actual statistical data of waste plastics cost in the enterprise, the cost of batching before and after optimization, is drawn as shown in Fig. 4.

Through comparison, it can be seen that the average cost and energy consumption after optimization by using NSGA II algorithm are obviously lower than that before optimization. After accounting, the application of NSGA II algorithm reduces the cost of ingredients by 6.42%, while the optimization effect of FP and AF algorithm is not obvious.

5. Conclusions

In this paper, the minimum batching cost is taken as the objective function, and the basic constraints and process constraints are taken into account to establish the batching model. On the premise of guaranteeing the normal chemical composition of polyester PET fibers, the cost per ton of polyester PET staple fibers has been reduced, and the optimization effect is remarkable. Numerous Pareto front-ends with uniform distribution can be obtained by using NSGA II algorithm. From my practical application in a chemical fibre factory, by inputting the stock material information into the database, the computer automatically calculates the optimal batching result according to the batching model, which is closer to the actual production than the simple linear programming batching model. The working effect is good and the production cost is reduced while ensuring the quality of polyester staple fibers. The model has strong practicability.
Acknowledgements

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References